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PREDICTING CLOSE APPROACHES AND ESTIMATING IMPACT PROBABILITIES FOR NEAR EARTH OBJECTS

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Recent popular movies have raised public consciousness of the very real possibility of a comet or asteroid collision with the Earth. A news story last year further caught the public's eye when it implied that asteroid 1997 XF_{11} had a distinct chance of hitting the Earth in the year 2028. The possibility of impact disappeared the very next day, and the public perceived either that astronomers had made mistaken calculations, or that the pre-discovery observations found that day had been responsible for the revised prediction. In fact, the original report of the possibility of impact in 2028 was due to an incomplete analysis. The XF_{11} affair has demonstrated the need for clarity and precision in public communications dealing with the possibility of Earth impact, as well as the importance of peer review before public statements like these are made. Recent analysis has shown that prior to the finding of the pre-discovery observations, there was a non-negligible probability that 1997 XF_{11} would collide with Earth many years after 2028. Another potentially hazardous asteroid, 1999 AN_{10}, has made the news this year, and this time there was indeed a remote chance of collision using the full data set. Although impact was never possible during this asteroid's primary close approach in 2027, the uncertainties allowed for a remarkably close passage. Embedded within the encounter's uncertainty region were many narrow "keyholes" which could bring the asteroid back for a close approach in a later year. Three keyholes were identified which could perturb the asteroid onto trajectories that collide with the Earth in the years 2044, 2046, or 2039. At one point, the estimated impact probability for 1999 AN_{10} was on the order of 1 in 500,000, larger than for any other known object, but still significantly less than the probability of an undiscovered asteroid of equivalent size striking the Earth before 2044. Additional astrometric measurements of 1999 AN_{10} later drove its impact probability down to near-zero. A side effect of the increasing discovery rate for Near Earth Objects will be a growing number of cases with at least temporarily non-negligible impact probabilities.

INTRODUCTION

Near-Earth Objects (NEOs) are asteroids and comets whose orbits bring them to the domain of the inner Solar System. These small bodies are of particular scientific interest because they are the leftover building blocks of the planetary formation process.

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But they are also worthy of our attention because of the hazard they pose to life on Earth. Even though the accretion phase of the Solar System ended long ago, as did the "heavy bombardment" phase, which produced the scars we see today on our Moon, Mercury, and other primitive bodies, the process of accretion and bombardment has not completely abated. Today, the Earth is still accumulating interplanetary material at the rate of about one hundred tons per day, although most of it is in the form of tiny dust particles released by comets as their dusty ices vaporize in the solar neighborhood.

The vast majority of the larger interplanetary material that rains down on the Earth originated as fragments from the collision of asteroids eons ago. Larger pieces of debris hit the Earth less frequently simply because there are fewer of them. Van-sized asteroids impact the Earth approximately every few years, but these typically disintegrate into small pieces before hitting the ground. Asteroids larger than about 50 meters, however, may well reach our surface largely in one piece, depending on their composition. Estimated to occur every few hundred years, an impact of a 100-meter-sized object would likely cause a local disaster, and if one occurred in the ocean, it could produce a tsunami that could inundate widespread low lying coastal areas. Asteroids larger than about one kilometer in size impact the Earth every few hundred thousand years or so, almost certainly producing a global catastrophe. Impact debris would rain down over a large fraction of the Earth's surface, producing widespread firestorms from the re-entry heating. Enormous quantities of fine impact debris would spread throughout the Earth's atmosphere, and global plant life would suffer from acid rain, and blocking of sunlight for months or years. Even though asteroid and comet impacts of this sort are extremely infrequent, the enormous consequences of these events make it prudent to mount efforts to discover and study these objects, to characterize their sizes, compositions and structures and to keep an eye upon their future trajectories.

Because of the ongoing search efforts to find nearly all the large NEOs, objects will occasionally be found on very close Earth approaching trajectories. Great care must then be taken to verify any Earth collision predictions that are made. Given the extremely unlikely nature of such a collision, almost all of these predictions will turn out to be false alarms. However, if an object is verified to be on an Earth colliding trajectory, it seems likely that this collision possibility will be known several years prior to the actual event. Given several years warning time, existing technology could be used to deflect the threatening object away from Earth.

NEO ORBIT DETERMINATION AND PREDICTION OF CLOSE APPROACHES

In order to evaluate the threat posed by known Near-Earth Objects, accurate orbits must be determined for each of them, and future close approaches must be predicted. Accurate orbits and close-approach predictions are also important in the planning of optical and ground-based radar observations, as well as for the design of spacecraft missions to these bodies. Optical astrometric observations provide the bulk of the data for orbit determination for asteroids and comets; each observation consists of the right ascension and declination of the object at a specified time. Modern optical astrometric

observations are typically accurate to better than one arc-second rms; the dominant error source is usually the star catalog used for the reference star coordinates. A second form of astrometric observation used for orbit determination is afforded by ground-based radar[1]. This highly accurate form of astrometric observation consists of measurements of echo delay and Doppler shift of a highly focussed signal transmitted to the object. The accuracy of the round-trip delay measurement typically corresponds to an accuracy of tens of meters in the range to the object.

The basic orbit determination problem is to estimate the six orbital elements $\mathbf{x}$ at some epoch t_0, given a set of astrometric measurements, typically numbering in the dozens or hundreds[2]. Once a preliminary orbit has been computed, usually from the first few of the observations, the problem becomes one of *orbit improvement* as additional observations are added to the data set. The problem is linearized about the previous best orbit estimate, and an improvement to the reference orbit is computed from the observation residuals (differences between the actual observations and predicted observations) via the linear method of least squares. The process is iterated until it converges, yielding the orbit estimate $\hat{\mathbf{x}}$ and the associated covariance matrix $\mathbf{P}$, which describes the accuracy of the estimate. At JPL, rather than using the standard normal-equation approach to solving the orbital least squares problem, we use a numerically more stable procedure called the square root information filter[3]. This method produces an upper triangular square root information matrix $\mathbf{R}$, from which the covariance matrix is computed via the relation $\mathbf{P} = \mathbf{R}^{-1}\left(\mathbf{R}^{-1}\right)^{T}$.

Asteroids and comets are often categorized according to their orbital character-istics. The customary definition of a Near-Earth Object is an asteroid or comet whose perihelion distance is less than 1.3 AU. The current number of known NEOs is about 800, although most of these are relatively small. If we restrict the count to NEOs larger than 1 km in size, the current number is about 300. Most of these objects cannot approach anywhere near the Earth except perhaps on million-year timescales. A more specialized category called the Potentially Hazardous Asteroids (PHAs) consists of those asteroids whose orbits approach within 0.05 AU of the Earth's orbit, and whose size is at least 150 meters or so. The current number of known PHAs is about 180. Figure 1 shows the orbit Potentially Hazardous Asteroid 1997 XF$_{11}$, which makes one of the closest predicted approaches to the Earth. The line of nodes is the intersection of the orbit plane of the asteroid with the ecliptic plane. Close approaches are possible when an object crosses through its line of nodes near the Earth's orbit.

Once the orbit for a NEO has been determined, its future close approaches to the Earth can be predicted by numerically integrating its position forward in time and monitoring the Earth-object distance. If the NEO has a secure orbit, we integrate as far forward as the year 2200. Objects with less secure orbits are integrated forward over shorter time spans, according to the data arc. The step size of the numerical integrator is adjusted automatically so as to maintain a local velocity error of less than 10^{-13} AU per day. Perturbations by the Earth and Moon are considered separately, rather than treating their combined masses as being located at their barycenter. Perturbations by the three

large asteroids Ceres, Pallas, and Vesta are included, and the general relativistic equations of motion are employed. When the integrator senses a planetary close approach, it interpolates within the trajectory to find the nominal close approach time to the one minute level, or better. (The actual uncertainty in the close approach time may be larger, due to the uncertainty in the object's initial orbital elements.) Close approaches to all perturbing bodies are detected, but only close approaches to the Earth are considered in this paper.

At JPL, we have implemented a semi-automated process to maintain an up-to-date database of PHA orbit solutions and close approach predictions. The process runs on a daily basis because it is essential to use the latest set of observations for each object, particularly when new data extend the data arc. Electronic circulars from the Minor Planet Center are checked daily for new PHA observations. If found, the new data are merged into the observational data set for the object, a new orbit is calculated, and future close approaches to the Earth are predicted. Our PHA orbital element tables and close-approach tables are available on our Near-Earth Object web site[4]. The current closest predicted Earth approach is that of asteroid 1999 AN_{10} in the year 2027, the miss distance being only 0.0026 AU. The 2028 encounter of 1997 XF_{11} is currently fourth closest, at 0.0064 AU.

LINEAR ANALYSIS OF CLOSE APPROACH UNCERTAINTIES

An object's orbit is never known exactly, because the orbit calculation is based on measurements which always contain small errors. The level of uncertainty in an orbit is quantified in the orbital covariance matrix P. Primary determinants of the orbital accuracy are (1) the time span over which the object was observed (referred to as the "data arc"), (2) the number and accuracy of the observations, (3) the object's proximity to the Earth when observed, and (4) whether or not radar observations were used in the orbit solution. The data arc is often characterized by the number of oppositions during which the object was observed, where an opposition is centered on a passage across the extended Sun-Earth line. If an asteroid has been observed over several oppositions, its orbit is typically quite well-determined, and its position can be predicted a decade into the future with high accuracy, usually better than a few arc-seconds on the plane of sky. Asteroids with such "secure" orbits are eligible for receiving an official number from the International Astronomical Union (IAU). The tally of such numbered asteroids is over 11,000. To date, only about 40 of the 180 PHAs have such secure orbits.

Asteroids with data arcs of fewer than three oppositions have less well-determined orbits, and are therefore generally not numberable unless they have been observed with radar. Single-apparition asteroids have only moderately well-determined orbits, at best. Still, if the data arc covers many months, reasonably accurate predictions can be made even a few decades into the future. But, if the data arc is less than a two months, the orbital accuracy deteriorates, and predictions beyond a decade or so are very uncertain. If the data arc is less than a week, predictions may not be accurate enough to ensure recovery during the next opposition, and the object may become lost. This is why follow-up observations in the weeks after discovery are so important.

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In the prediction of future close approaches of NEOs it is important to consider the uncertainties in the close approach circumstances, uncertainties which can be quite large. In particular, the impact probability is a function of the position and velocity uncertainties during the encounter. The uncertainties can be estimated using a linear covariance procedure which maps orbit uncertainties at a contemporary epoch to position and velocity uncertainties at the predicted time of close approach. The mapping is accomplished by computing the state transition or mapping matrix $\mathbf{Y}(t) \equiv \partial \mathbf{r}(t)/\partial \mathbf{x}$ at the close approach time t_{CA}, where $\mathbf{r}$ is the heliocentric position vector. We compute this matrix via numerical integration of the so-called variational equations,

$$\ddot{\mathbf{Y}}(t) = \frac{\partial \ddot{\mathbf{r}}(t)}{\partial \mathbf{r}} \mathbf{Y}(t) + \frac{\partial \ddot{\mathbf{r}}(t)}{\partial \mathbf{v}} \dot{\mathbf{Y}}(t) ,$$

where the partials of $\ddot{\mathbf{r}}(t)$ are computed analytically. The variational equations are numerically integrated at the same time as the equations of motion.

For our close approach computations we use the so-called b-plane, or *impact plane*, the plane perpendicular to the incoming asymptote of the hyperbolic geocentric trajectory. The impact plane is superior to the *target plane*, the plane perpendicular to the velocity vector at close approach, because the latter is more susceptible to the problem of differential perturbations over extended regions of the plane. The standard set of b-plane elements uses the geocentric position $\mathbf{b}$ of the intercept of the incoming asymptote with the impact plane, which is equivalent to the close approach position vector if the Earth were massless. We use a variation of the standard approach, by defining a *scaled* impact parameter $\mathbf{b}_p$ given by

$$\mathbf{b}_p = \frac{1}{v_p} \mathbf{s} \times \mathbf{h} ,$$

where $\mathbf{s}$ is the unit vector along the incoming asymptote, $\mathbf{h} = \mathbf{r} \times \mathbf{v}$ is the specific angular momentum vector. The parameter v_p is the velocity at the planet's surface, given by

$$v_p = \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a}} ,$$

where r_p is the radius of the planet, μ is the gravitational parameter of the planet, and a is the semi-major axis of the object's two-body trajectory relative to the planet. Because of our use of v_p instead of the hyperbolic excessive velocity v_∞, the figure of the Earth in the impact plane is a disk of radius r_p, not the capture radius r_c. The square root covariance of the uncertainty in the scaled b-plane elements $\mathbf{x}_{sb}$ is given by

$$\mathbf{S}_{sb} = \left(\frac{\partial \mathbf{x}_{sb}}{\partial \mathbf{r}} \mathbf{Y} + \frac{\partial \mathbf{x}_{sb}}{\partial \mathbf{v}} \dot{\mathbf{Y}} \right) \mathbf{R}^{-1} .$$

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The upper left 2×2 partition of $P_{,b} = S_{,b} S_{,b}'$ describes the uncertainty in b_p, a 2-dimensional marginal Gaussian probability density function displayed graphically as an uncertainty ellipse. The probability of impact is estimated by integrating this marginal probability density function over the Earth disk via an efficient semi-analytic technique[5]. The third component of our scaled b-plane elements is the linearized time of flight, and its uncertainty represents the uncertainty in the close approach time. In our close approach tables we include this uncertainty in the close approach time, as well as the minimum three-sigma close approach distance, the number of sigmas by which the uncertainty ellipse must be scaled to yield a grazing impact, and the impact probability.

THE CASE OF ASTEROID 1997 XF$_{11}$

In March 1998, asteroid 1997 XF$_{11}$ received a great amount of press attention because of the prediction that it could make an extremely close approach to the Earth in 2028, and the suggestion that a collision was a distinct possibility. The asteroid had been discovered on December 6, 1997 by Jim Scotti using the Spacewatch Telescope on Kitt Peak, and had been placed on the Minor Planet Center's list of Potentially Hazardous Asteroids soon afterwards. After a month, its orbit was well enough determined for the Center to predict that the asteroid would pass within a million kilometers of the Earth on October 26, 2028. The asteroid was well observed for another month, but then went unobserved for four weeks. When Peter Shelus at the McDonald Observatory in Texas picked it up again on the nights of March 3 and 4, his four observations extended the data arc significantly to 88 days, and yielded a significantly improved orbit estimate. On March 11, Brian Marsden, director of the Minor Planet Center, announced in an IAU Circular[6] that the new prediction for the miss distance in 2028 was remarkably small, less than a quarter of a lunar distance. This was easily the closest approach of a sizeable asteroid to the Earth ever predicted. Based on its magnitude, the object was estimated to be over a kilometer across, large enough to cause a global catastrophe if it should impact, and it would be fairly bright if it did indeed come as close to Earth as predicted. The Circular went on to say that "error estimates suggest that passage within [one lunar distance] was virtually certain". In an accompanying press statement, Marsden stated "The chance of an actual collision is small, but one is not entirely out of the question." Although no specific year was included in this statement, the clear implication was that a collision in 2028 was distinctly possible, and that is how the press and media interpreted the story. Asteroid 1997 XF$_{11}$ became front-page news worldwide.

Unfortunately, a full covariance analysis of the close approach uncertainties had not been carried out before the above statements were released to the press. We carried out such a linear analysis only hours after receiving the March 11 press release and determined that an impact in 2028 was essentially impossible. (In fact, the formal impact probability computation yielded a result of precisely zero, which indicates a value less than the underflow limit of the machine, approximately 10^{-300}; later independent analyses support this conclusion[7].) Figure 2 shows the 3-sigma position uncertainty ellipse in the target plane at closest approach (sigma denotes standard deviation). The ellipse is extremely elongated, about 2.8 million kilometers long, but only 2,500 km wide. The

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extreme length of the ellipse is due to the fact that the position uncertainty along the orbit grows linearly with time over the 30-year prediction period, while uncertainty perpendicular to the orbit varies only periodically. (It is interesting to note that the 30-year projection into the future spans 17 revolutions of the asteroid about the Sun.) Since the ellipse extends well beyond the Moon's orbit, passage outside one lunar distance is very possible. The great length of the ellipse in Figure 2 makes it difficult to predict a precise miss distance, because passage virtually anywhere within the ellipse is possible, according to the observations. But the surprisingly narrow width of the ellipse allows a fairly precise determination of the *minimum possible miss distance*, about 28,000 km. Figure 3 shows a close up of the region of the target plane near the Earth. The ellipse would have to be enlarged to about the 55-sigma level before it would include orbits grazing the Earth. In fact, the impacting orbit solution which best fits the 98 observations had residuals running from +10 arc-sec to −10 arc-sec, with root-mean-square residual of 4 arc-sec, clearly an impossibly bad fit to the data.

On the very next day, March 12, Ken Lawrence and Eleanor Helin, both of JPL, found four pre-discovery images of the asteroid, taken in 1990, greatly extending the data arc to 8 years. We computed a new orbit solution essentially at the same time as the Minor Planet Center, and both groups reached the same conclusion: the revised prediction for the close approach in 2028 moved out to a much less impressive 980,000 km, over two lunar distances from the Earth. Our updated linear analysis of the close approach uncertainty showed that the ellipse shrank in length by over an order of magnitude. Figure 4 shows the uncertainty ellipse for the orbit solution including the 1990 observations. The minimum possible close-approach distance increased to a comfortable 865,000 km, and the probability of impact remained essentially zero. Unfortunately, most press reports on this second day were terribly inaccurate, for example, crediting the new observations for ruling out the possibility of collision. In reality, *there had never been a distinct possibility of collision in 2028.*

NON-LINEAR MONTE CARLO ANALYSIS OF CLOSE APPROACHES

Linear covariance methods for computing close approach uncertainties can break down if the position uncertainty grows too large, e.g. if the prediction period is long, or the object makes an extremely close approach to a planet. As the uncertainties increase, the linearity assumption becomes less and less tenable. Milani and Valsecchi propose a semi-linear method for computing target plane confidence boundaries, in which marker points are distributed around the linear ellipse, linearly mapped back to initial conditions at epoch, and then integrated forward to close approach using the non-linear equations[8]. A more rigorous but computationally intensive method for analyzing orbital uncertainties uses a Monte Carlo approach [9,10]. With this technique, the six-dimensional hyper-ellipsoid representing the uncertainty of the orbital elements at epoch is populated with thousands of random test points to obtain an ensemble of initial conditions consistent with the 6×6 covariance matrix of the orbit solution. The test points are all integrated forward using the fully nonlinear equations of motion, the close approaches of each case are recorded, and statistical analyses are performed on the resulting ensemble of close approaches. Dynamical non-linearities are fully taken into account with this approach,

but a large number of points must be used to sample adequately the ellipsoid of initial conditions, and the computational burden is therefore considerable. The advent of powerful and cheap microcomputers, however, makes the Monte Carlo technique quite feasible.

The procedure for creating the ensemble of Monte Carlo initial conditions is particularly simple when the orbit is determined using the square root information filter. If $\mathbf{w}$ is a zero-mean, unit-standard-deviation random 6-vector, then $\hat{\mathbf{x}} + \mathbf{R}^{-1}\mathbf{w}$ is a random 6-vector with the proper mean and covariance to represent an initial condition. Statistics are kept on these initial conditions, and as the number of cases increases, their mean and covariance approach those of the orbital solution. As each test case is integrated forward in time, encounters within specified distances of all perturbing bodies are monitored, and the position and velocity of the test case relative to the target body are recorded at the moment of each close approach.

In order to perform a two-dimensional statistical analysis of the close approaches, they must be projected into a common plane, and we again choose the impact plane for this purpose. As with the linear methods described above, we use a variation of the standard approach, computing the scaled impact plane coordinates from

$$b_{pt} = -\frac{1}{v_p} \hat{\mathbf{r}}^T \mathbf{h},$$

$$b_{pr} = \frac{1}{v_p} \hat{\mathbf{t}}^T \mathbf{h},$$

where $\hat{\mathbf{r}}$ and $\hat{\mathbf{t}}$ are basis vectors in the impact plane, and $\mathbf{h} = \mathbf{r} \times \mathbf{v}$ is the specific angular momentum vector for the test case. To ensure best accuracy for cases approaching closest to the target body, the impact plane is computed not from the nominal case, but from the case which makes the closest approach to the target body. As described earlier, the figure of the Earth in the scaled impact plane is a disk of the planetary radius r_p. A straightforward method of computing the impact probability from the ensemble of Monte Carlo cases is simply to count the number of cases with scaled impact coordinates within one planetary radius of the origin. An improved method will be discussed later.

It is instructive to apply the non-linear Monte Carlo close approach analysis to the 2028 close approach of 1997 XF$_{11}$ and compare with the linear analysis described earlier, which also used the scaled b-plane approach. Figure 5 shows a close-up of the portion of the ellipse closest to the Earth, with the Monte Carlo cases overlaid, and the good agreement is evident. The closest approach of 600,000 cases was 28,034 km, consistent with the prediction based on linear theory. On the other hand, Figure 6 shows one of the ends of the ellipse, and there is clearly an offset between the major axes of the linear ellipse and non-linear Monte Carlo points, although the two axes are only displaced by about 2000 km. One could conceive of pathological cases with the Earth placed near the end of the linear ellipse in such a way that linear analysis yields grossly incorrect results,

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but in this particular case, with the Earth near the center of the ellipse, the linear method performed adequately. Clearly, however, the Monte Carlo method is to be preferred if time allows a sufficient number of cases to be run.

COULD ASTEROID 1997 XF$_{11}$ COLLIDE AFTER 2028?

Three months after the 1997 XF$_{11}$ story first hit the news, Marsden announced on an Internet discussion group evidence that, *prior to the discovery of the 1990 observations*, "there was in fact a small, but real, possibility of collision" in the decade or so *after* 2028. He noted that the asteroid's descending nodal crossing was outside the Earth's orbit in 2028, but perturbations would cause the nodal distance to decrease in subsequent years, and, in fact, cross the Earth's orbit around 2037. Furthermore, the close approach in 2028 could change the orbital period of the asteroid from 1.73 years to anything between 1.58 and 1.99 years, depending on the geometry of the encounter. For every rational number between these limits there is a corresponding trajectory which would bring the asteroid back for another close approach to the Earth in a later year. For example, if the asteroid passed about 210,000 km from the Earth in 2028, its period would change to 1.80 = 9/5 years, which would bring it back to Earth in 2028 + 9 = 2037, during which time the asteroid would complete 5 orbits. Marsden provided an example orbit, consistent with the 88-day set of observations, which he claimed would actually impact in that year. These new results did not dispel criticism of Brian's original press statement on the chances of collision (for that statement clearly implied the 2028 close approach), but they did open a new area of investigation.

Linear methods were clearly inadequate to investigate the post-2028 close approach uncertainties based on the 88-day-arc solution, because the uncertainties after 2028 were much too large. The day after Marsden's article, we performed a preliminary analysis of the 2037 close approach using essentially the same non-linear Monte Carlo technique as described above. Our conclusions disagreed with Marsden's: we found that an impact in 2037 was essentially impossible, but an extreme close approach of only 0.5 Earth radius above the surface was possible. In a June 10th e-mail to our colleagues summarizing our results, we noted that "about 40 cases out of 20,000 passed through a *keyhole* in the 2028 ellipse to arrive within 1.5 million km of Earth in 2037". The descriptive term "keyhole" refers to a narrow linear slice of the uncertainty ellipse where the encounter can perturb the object onto a trajectory which returns for a later close approach. The keyhole to the 2037 encounter did not seem to allow an impact in that year, but what about keyholes to close approaches in other years? Marsden later identified another impacting scenario based on the 88-day arc, this one having a post-2028 period of 12/7 = 1.71 years, and leading to Earth impact in 2040. His estimate for the impact probability was as high as 1 in 100 thousand[11].

We have performed an extensive non-linear Monte Carlo uncertainty analysis of all close approaches of 1997 XF$_{11}$ over the next 50 years, using the orbital solution based on the 88-day data arc. Our analysis confirms that an impact in 2040 was indeed possible, and, further, that the impact probability was non-negligible in 4 other years, as indicated in Table 1. The impact probability for 2040 was at least an order of magnitude more

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likely than that for any other year. Note also that this analysis confirmed our preliminary assessment that an impact in 2037 does not seem possible. The total chance of collision over the 50-year period was on the order of a few parts in 10^5.

Table 1
POST-2028 PROBABILITY OF IMPACT FOR 88-DAY ARC OF 1997 XF$_{11}$

Year	Probability of Impact
2040	2×10^{-5}
2041	2×10^{-6}
2043	2×10^{-6}
2048	1×10^{-6}
2039	7×10^{-7}

Figure 7 shows the Monte Carlo cases plotted on the impact plane for the 2040 encounter. Points lying within the central circle represent Earth impacts: 12 cases impacted out of 600,000. When traced back to the 2028 uncertainty region, we find that these points passed through an impact keyhole about 100 km wide, as shown in Figure 8. All of the points from the main stream in Figure 7 passed through a broad 'encounter' keyhole centered on the impact keyhole in Figure 8. The two points *below* the main stream in Figure 7 arrived via a secondary keyhole in the 2028 uncertainty region, a keyhole much closer to the Earth, as shown in Figure 9. A linear interpolation between the initial conditions for these two points yielded close approaches intermediate between the two points in the impact plane, showing that they lie on a single highly stretched stream. Since the stream passes about 14,000 km from the Earth center, we conclude that points on this stream cannot impact the Earth.

The impact plane for the 2041 encounter is shown in Figure 10. In this case, four streams are evident, two main streams corresponding to keyholes which produce orbit periods near 13/7 and 13/8 years (both periods lying within the dispersion capability of the 2028 close approach), and the respective secondary keyholes near the Earth in 2028. One main stream clearly crosses the Earth, while the other just grazes; a highly stretched secondary stream also crosses the Earth. The impact probability for stretched streams such as this one can be estimated by computing the linear density of points over a domain centered on the origin, and multiplying by the chord length of the stream's Earth crossing. For the year 2041, the impact probability is dominated by the contribution from the main stream.

As a postscript, we performed a similar non-linear Monte Carlo analysis for the current orbit solution for 1997 XF$_{11}$, using the full set of observations. Because of the 1990 data, this orbit has significantly smaller position uncertainties to begin with, and its much shallower close approach in 2028 does not magnify the uncertainties nearly as much. We found no impacting cases for at least the next hundred years, with the closest possible approach occurring in 2083 at a distance of 135,000 km. We conclude that 1997 XF$_{11}$ has no significant chance of colliding with Earth for at least a century.

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THE CASE OF ASTEROID 1999 AN₁₀

In April 1999, another potentially hazardous asteroid, 1999 AN_{10}, made the news because of a remote possibility that it might collide with Earth. The story of this asteroid is remarkably similar to that of 1997 XF_{11}, except that, fortunately, no incorrect claims were made that an impact was possible during the asteroid's first deep approach to the Earth. The asteroid was discovered on January 13, 1999 by the Lincoln Near-Earth Asteroid Research (LINEAR) program operated by MIT's Lincoln Laboratory in cooperation with the U.S. Air Force. Based on its brightness, the object was estimated to be just over one kilometer in size, close to the most dangerous size range. The asteroid's orbit is somewhat unusual because it passes fairly close to the Earth's orbit not just once, but twice on each 643-day circuit about the Sun, both inbound towards the Sun and outbound from the Sun.

Unfortunately, 1999 AN_{10} was observed for less than 6 weeks before it moved into the glare of the daytime sky, and because this data arc was so short, predictions of the asteroid's motion were very uncertain. A deep close approach to the Earth was possible on August 7, 2027, but the uncertainty region for this close approach was much larger even than that for the 2028 encounter of 1997 XF_{11} before its pre-discovery observations were found. The minimum possible close approach distance for 1999 AN_{10} was a little larger, about 38,000 km from the center of the Earth -- an impact was therefore not possible in 2027. But what about impacts after 2027?

During this time, Andrea Milani and his colleagues in Italy had been independently developing non-linear techniques for analyzing close approaches, with the idea of applying these to the case of 1997 XF_{11}. When these methods were applied to the case of 1999 AN_{10}, it was found that one of the predicted close Earth approaches could result in an impact, with a probability on the order of one in a billion. In late March, Milani *et al.* circulated a preprint which announced this remote possibility of impact for 1999 AN_{10}, and outlined a theory which predicted both its resonant and non-resonant returns[12,13]. (Non-resonant returns referred to trajectories which took the asteroid from an encounter at one node to an encounter at the other node.)

The impacting scenario identified by Milani *et al.* was for the year 2039, but it actually required that 1999 AN_{10} pass through two keyholes, one in the 2027 confidence region, which would take it to a 2034 close approach, and then through a second keyhole in the 2034 confidence region. This partly explained why this scenario was so unlikely: At one-in-a-billion, this impact probability was so extremely miniscule, that it was tens of thousands of times smaller than the probability of an undiscovered asteroid of equivalent size hitting the Earth during the same 40-year period. However, the asteroid certainly deserved to be watched carefully, as Milani *et al.* had found that its orbit would remain threateningly close to the Earth's orbit for many centuries to come. Fortunately, in just a few months, the asteroid would become observable again, as it moved back into the twilight sky. It was expected that the new observations would, in all likelihood, completely eliminate the possibility of impact in 2039.

New observations for 1999 AN$_{10}$ were indeed made in mid-May, 1999, by amateur astronomer Frank Zoltowski in Australia. As expected, these enabled much more precise orbital calculations, and the revised predictions indicated that the asteroid was even more likely to make a particularly close passage of the Earth on August 7, 2027. The minimum possible approach was just 37,000 km from the Earth's center (just 19,000 miles above the surface), but, as with 1997 XF$_{11}$, the asteroid could just as easily pass outside the Moon's orbit. Figure 11 shows the uncertainty in the predicted close approach in 2027, based on the 123-day data arc available in mid-May, 1999. As before, the uncertainty ellipse is so extremely elongated that it appears as just as a line segment. To be precise, the ellipse as drawn is actually the three-sigma linear confidence boundary. The center of the ellipse is indicated by the plus sign, located at a nominal distance of 58,000 km from the center of the Earth. The position of the keyhole which led to the possible impact in 2039 is shown at the right end of the uncertainty ellipse. This impacting scenario was still possible, and in fact had become about 100 times more likely, since the new uncertainty ellipse had shrunk by a factor of about 100 from its original size in March.

With the new orbit still indicating a deep encounter in 2027, it could be expected that many more keyholes might exist in the uncertainty region for 1999 AN$_{10}$. Milani's group in Italy and our group at JPL simultaneously performed new non-linear analyses of the close approach uncertainties, and identified two new impacting possibilities for the years 2044 and 2046. Each required passage through only a single keyhole in the 2027 uncertainty region, and the probabilities of impact for these cases were correspondingly larger than that for 2039. The estimated impact probability for the year 2044 was on the order of 1 in 500,000, and for the year 2046, about 1 in five million. These odds of collision were larger than those for any other object, but they were still only one hundredth the chance of an undiscovered asteroid of equivalent size striking the Earth sometime before 2044. Figure 12 shows the keyhole in the 2027 uncertainty region which leads to impact in 2044; it was only about 5 km wide. In the impact plane for the 2044 encounter, the Monte Carlo cases break down into two streams, as shown in Figure 13. Impacts are clearly possible on the upper stream.

As additional observations became available in late May and June, the orbit was refined yet again. The uncertainty region shrank somewhat, and moved completely off the 2039 keyhole, which indicated that this impacting scenario was no longer possible. Then, in July, two German amateurs looking through digitized plate catalogs found pre-discovery images of 1999 AN$_{10}$ on archival plates taken in 1955 for the Palomar Sky Survey. Just as was the case for 1997 XF$_{11}$, the pre-discovery data enabled the calculation of a greatly improved the orbit, and the uncertainties shrank dramatically. Fortunately, the new uncertainty regions did not enclose the keyholes which could lead to impact, and probability of impact for 1999 AN$_{10}$ also shrank, to essentially zero.

ACKNOWLEDGEMENT

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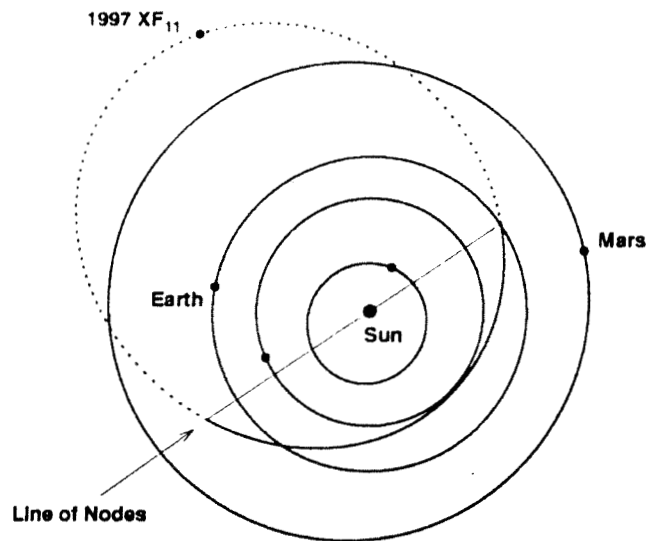
Figure 1 Orbit of Asteroid 1997 XF₁₁
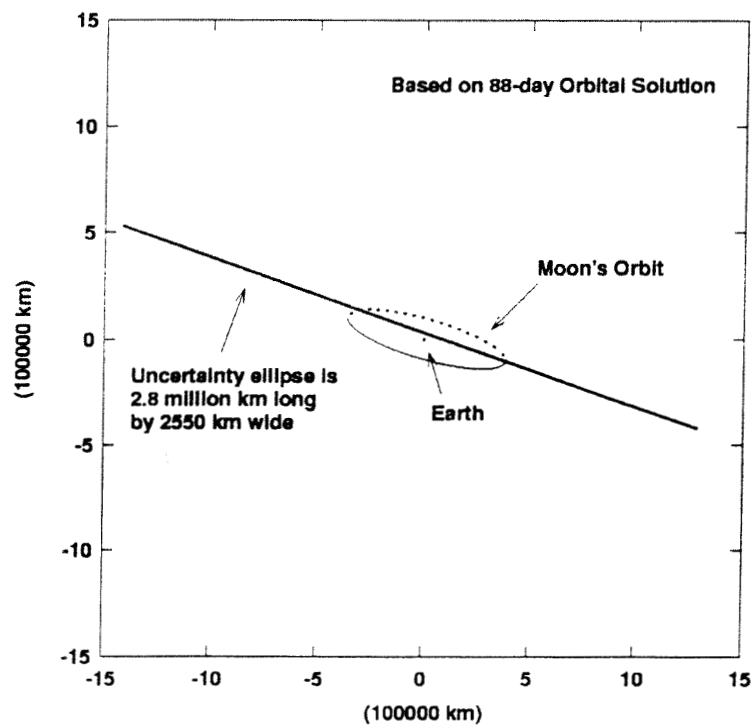**(Orbit inclined 4.1° to ecliptic plane; dotted portion below ecliptic)**

Figure 2 Position Uncertainty Ellipse for 1997 XF₁₁ in Target Plane on
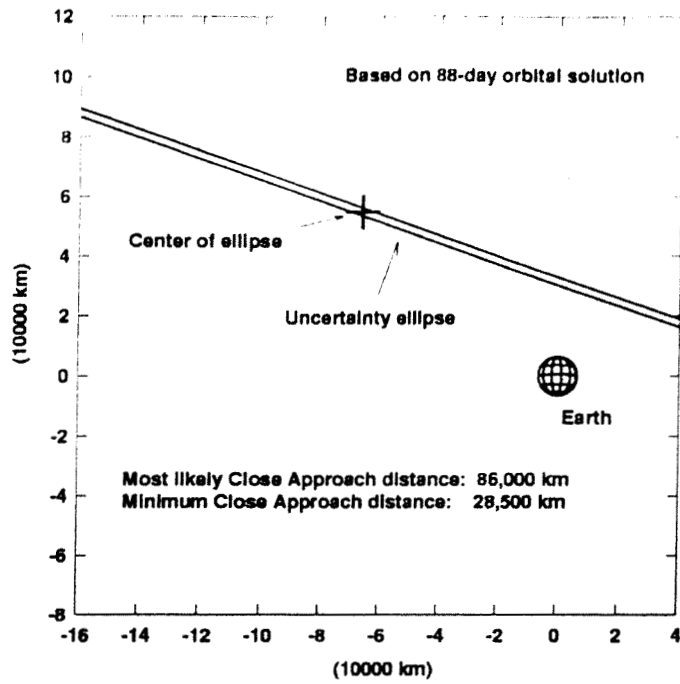**Oct. 26, 2028, 88-day Orbital Solution**

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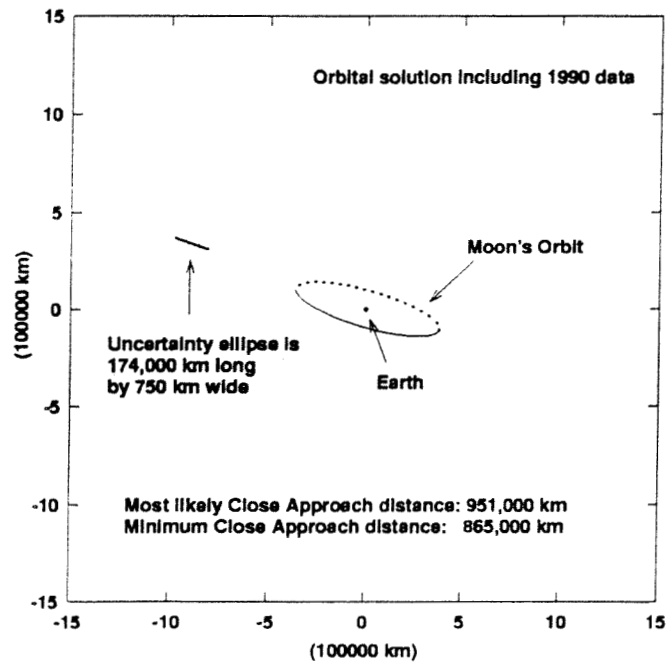**Figure 3 Enlargement of Central Part of Figure 2**

Figure 4 Position Uncertainty Ellipse for 1997 XF$_{11}$ in Target Plane on Oct. 26, 2028, Orbital Solution Including 1990 Observations
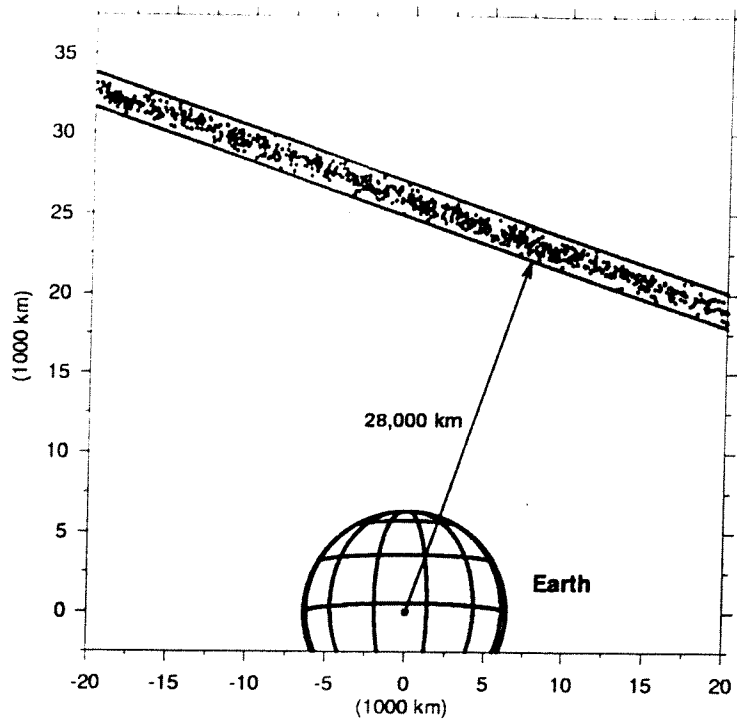
Figure 5 Monte Carlo points and Uncertainty Ellipse for 1997 XF₁₁ in 2028 Impact Plane, 88-day Orbital Solution
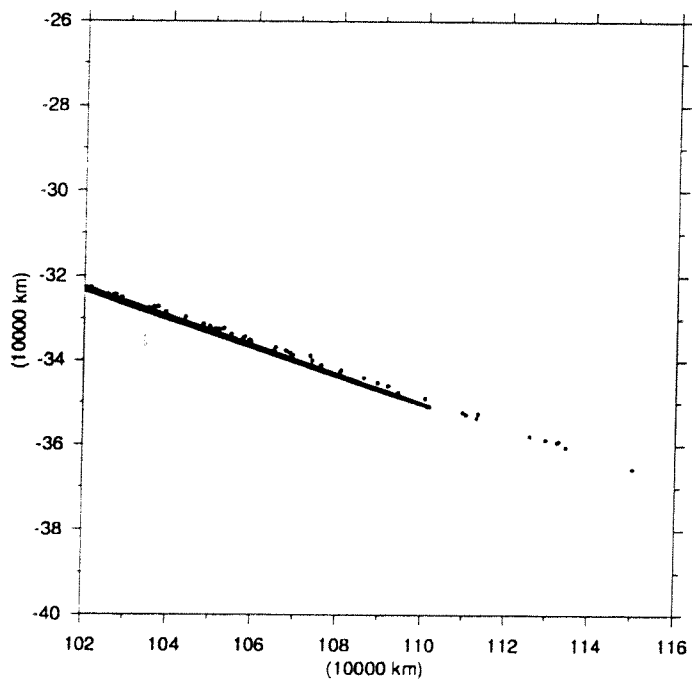
Figure 6 Monte Carlo points and Uncertainty Ellipse for 1997 XF₁₁ in 2028 Impact Plane, 88-day Orbital Solution
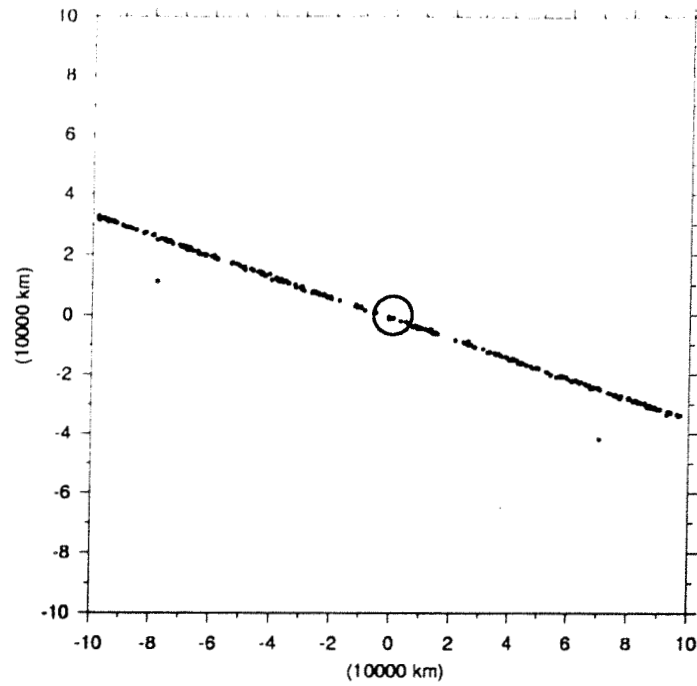
Figure 7 Monte Carlo points for 1997 XF$_{11}$ in 2040 Impact Plane, 88-day Orbital Solution
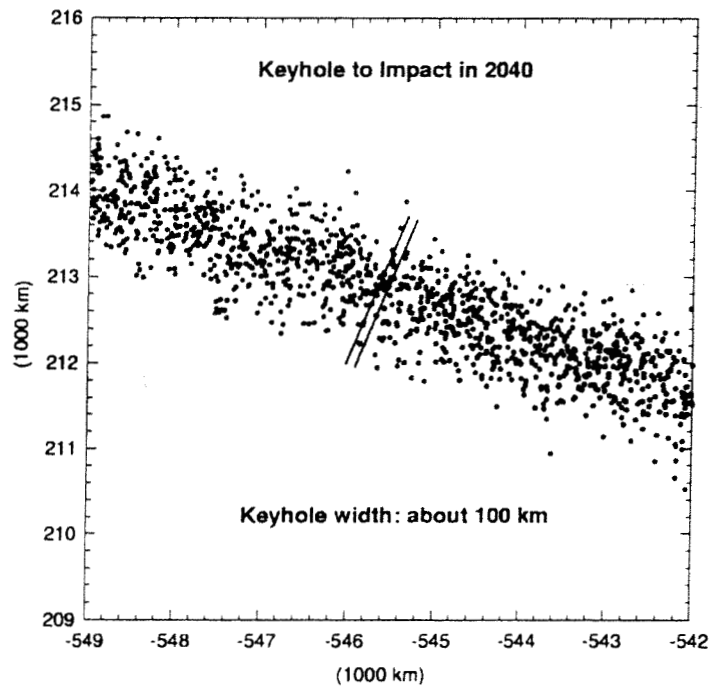
Figure 8 2028 Impact Plane for 1997 XF$_{11}$, 88-day Orbital Solution, Keyhole to Impact in 2040
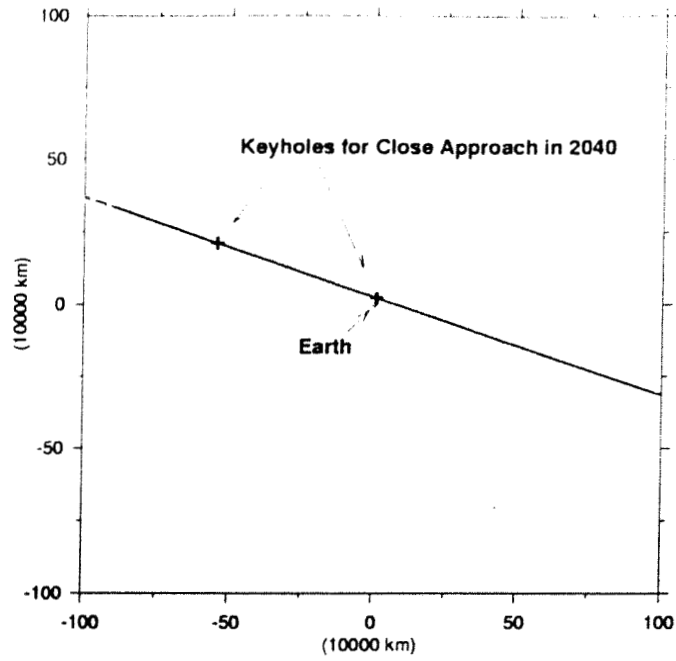
Figure 9 Uncertainty Region for 1997 XF$_{11}$ in 2028 Showing Positions of Keyholes to 2040 Close Approaches
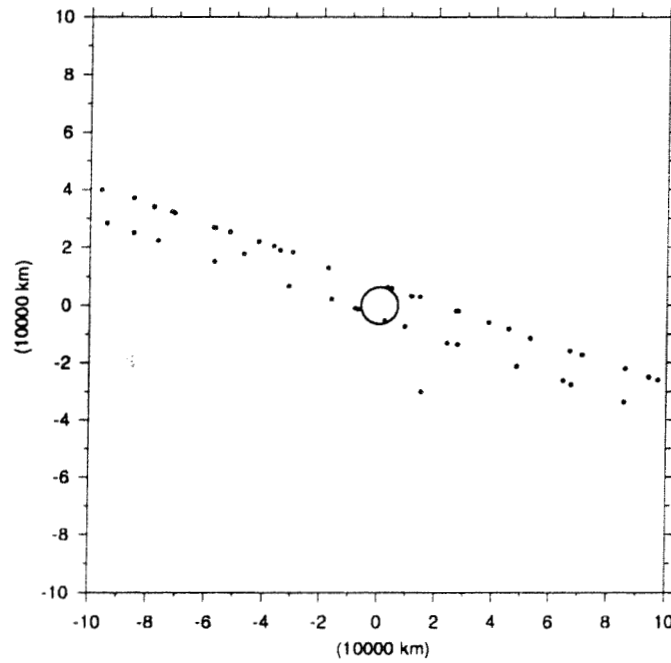
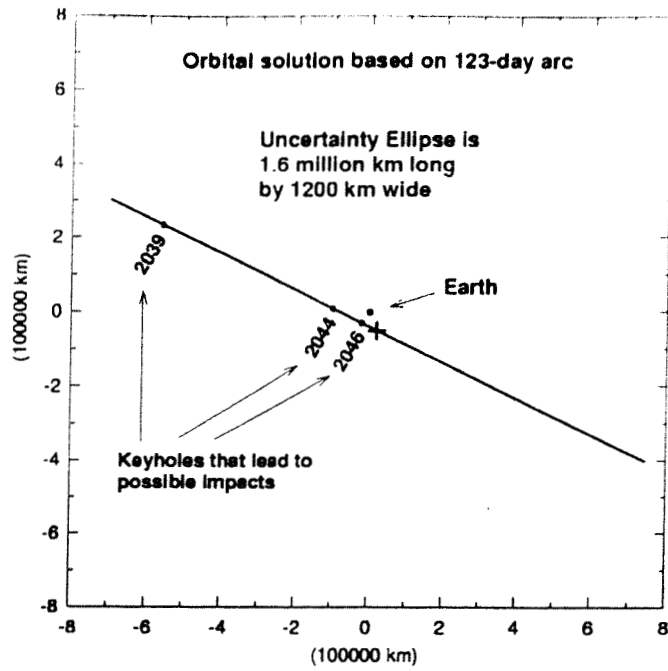**Figure 10 Monte Carlo points for 1997 XF$_{11}$ in 2041 Impact Plane, 88-day Orbital Solution**

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Figure 11 Uncertainty Ellipse for 1999 AN₁₀ in 2027 Impact Plane
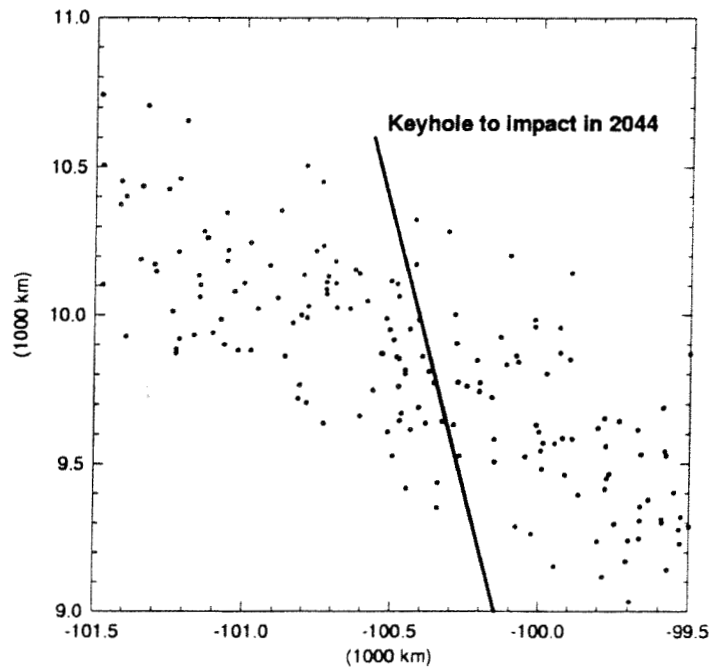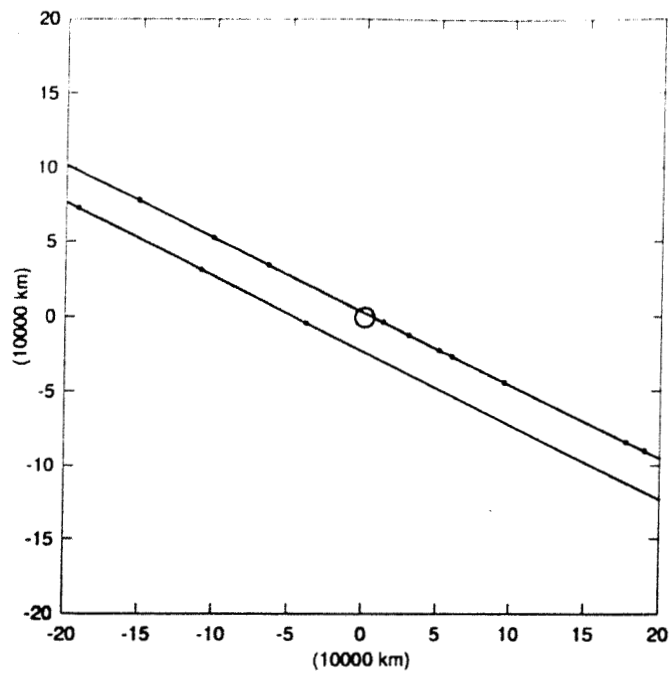

**Figure 12 2027 Impact Plane for 1999 AN₁₀,
Keyhole to Impact in 2044**

Figure 13 2044 Impact Plane for 1999 AN$_{10}$